EXAMPLE 2

To prove this theorem we will use the commutative and associative properties to show that $(ab) \cdot \left(\frac{1}{a} \cdot \frac{1}{b}\right)$ is 1.

Proof

1. $(ab)\left(\frac{1}{a} \cdot \frac{1}{b}\right) = \left(ab \cdot \frac{1}{a}\right) \cdot \frac{1}{b}$	1. Associative property of multiplication
2. $= \left(a \cdot \frac{1}{a} \cdot b\right) \cdot \frac{1}{b}$	2. Commutative property of multiplication
3. $= \left(a \cdot \frac{1}{a}\right)\left(b \cdot \frac{1}{b}\right)$	3. Associative property of multiplication
4. $= 1 \cdot 1$	4. Definition of reciprocal
5. $= 1$	5. Multiplicative identity
6. $ab\left(\frac{1}{a} \cdot \frac{1}{b}\right) = 1$	6. Transitive property of equality

Thus $\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}$.

This theorem gives us the procedure for multiplying two fractions or two rational expressions.

Theorem

For any rational numbers a, c, and any nonzero rational numbers b, d,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Try This

a. Complete the following proof of the theorem presented above.

Proof

1. $\frac{a}{b} = a \cdot \frac{1}{b}$ and $\frac{c}{d} = c \cdot \frac{1}{d}$	1. Division theorem
2. $\frac{a}{b} \cdot \frac{c}{d} = \left(a \cdot \frac{1}{b}\right)\left(c \cdot \frac{1}{d}\right)$	2. Substituting $a \cdot \frac{1}{b}$ for $\frac{a}{b}$ and $c \cdot \frac{1}{d}$ for $\frac{c}{d}$
3. $= (a \cdot c)\left(\frac{1}{b} \cdot \frac{1}{d}\right)$	3.
4. $= ac\left(\frac{1}{bd}\right)$	4.
5. $= \frac{ac}{bd}$	5.
6. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	6.

b. The subtraction theorem states that for any real numbers a and b, $a - b = a + (-b)$. Write a proof of the subtraction theorem.